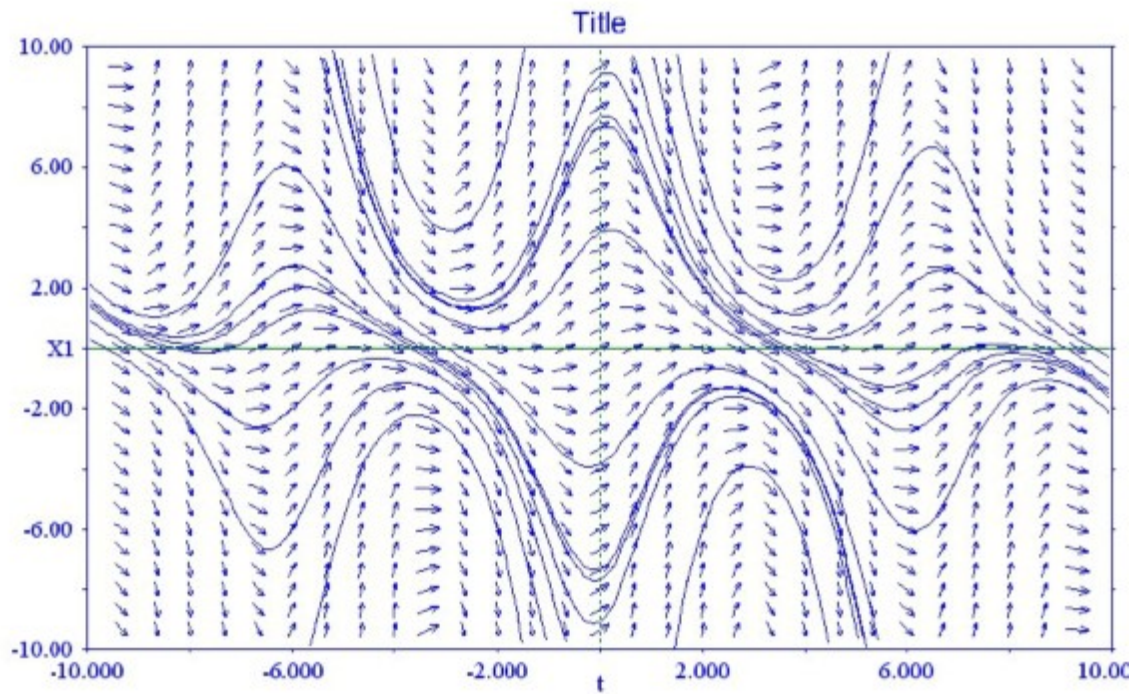


## DEQ For Windows

AN INTRODUCTION TO USING DEQ  
A DIFFERENTIAL EQUATIONS SOLVER



(Revised Jan 2016)

DEQ is a differential equations solver. It employs various numerical methods for evaluating and graphing solutions of systems of first order differential equations and equations which can be written as systems of first order differential equations. It will solve a system of up to five first order equations. The differential equations can include up to six parameters whose values can be changed to produce different solutions to the same family of differential equations.

When you type in your differential equation(s), you will need to express your variables as follows:

1. You may use lower case or uppercase letters.
2. The independent variable must be t.
3. The dependent variables must be x1, x2, x3, x4, x5, depending on the number of equations you have.
4. The parameters, if you have any, must be a,b,c,d,e,f, depending on the number you use. You should start with the letter a first and proceed in alphabetical order.

For our first example, let's solve the differential equation  $\frac{dy}{dx} = -\sin(x)y + \cos(x)$

Click on Equations. You will see  $X1' =$  . You just need to enter the right side of the differential equation. You will replace the independent variable x with t and the dependent variable y with x1. Your equation will then look like

$-\sin(t)*x1 + \cos(t)$  so type this in for your equation. [\* means multiplication]

To set up a graphing window, Click on Axes and Change Axes and type in Mint=-10, Maxt=10, MinX1=-10, and MaxX1=10. Note that your axes are labeled X1 and t.

We're now ready to draw some solutions and visualize their behavior..

1. Click on Direction Field. The window will be filled with a grid of points (t,x1) at which short tangent line segments are drawn. If a solution to the D.E. passes through that grid point, the line segment indicates the direction of the tangent line at that point. The direction field "suggests" how the solutions to the D.E. behave.
2. Click on Initial Conds. Type in 1, so the initial condition is  $x1(0)=1$ .

Click on Draw to plot the solution or trajectory which satisfies your initial condition  $x_1(0)=1$ . The solution has been plotted on the  $t$ -interval from  $t=0$  to  $t=10$ , which is the default  $t$  interval for the window size you selected.

3. Click on Edit|Backward This choice means that instead of plotting the solution curve on the time interval from  $T=0$  to  $T=10$ , you can now plot the solution curve on the interval from  $T=-10$  to  $T=10$ . This can be helpful if you want to see where the solution came from before it arrived at the initial point  $x_1(0)=1$ .

Now Draw the solution again. This time, the trajectory will be extended to the time interval from  $t=-10$  to  $t=10$ .

4. You may view the points  $(t, x_1)$  that were computed by the Runge-Kutta fourth order method for the last trajectory you plotted. Only forward values are shown. To do this, Click on File|List Solution Values . This data list provides you with the actual approximated values of the solution through the current initial value. If you want hard copy, you can select Print to print the data to the printer. Be sure the Printer is on, has paper, and is ready to go. You can also edit the list of values by cutting away some of the data before you print, so you don't get loads of pages of numbers.

Also, notice that in this list of data, the solution values are given at every tenth of a unit in the  $t$ -column. That's because the step-size in the Runge-Kutta scheme has a default value of 0.1. This value can be changed by clicking on Edit|Stepsize in the Main Menu. If you encounter trajectories that are "jagged" or rough-looking, you can choose a smaller step size, such as 0.05. A smaller step size means more points are plotted and plotting time increases. Generally, the default step size works on most D.E.'s.

5. Click on Samples. This option applies only to a single first order D.E. It establishes a grid of initial points  $(t, x_1)$  in the window, and then uses Euler's Method to quickly plot a sample of trajectories through each initial point  $(t, x_1)$  both backwards and forward. The solutions should follow the direction field tangent segments, which you can draw again by just clicking on Direction Field . Euler's Method is generally not as accurate as Runge-Kutta, but for this example, the results are pretty good.

6. Click on Edit|ClearScreen to Clear the graphics screen.

7. Plot up a variety of trajectories by clicking on Initial Conds and entering a different initial value, then Draw to plot the trajectory through that initial point.

Another way to plot up lots of solutions quickly is to press the right mouse button, move the cursor to an initial point, and release the button. The solution is plotted and the coordinates of the initial point are recorded on the track-bar to the right of the graphing window.

8. The selection Axes allows you to move the graphics window to another location. Click on Axes|Change Axes and type in Mint=-1, Maxt=20, MinX1=-15, and MaxX1=15. The new graphing window will appear, and you can begin drawing some solutions in this new part of the plane, or draw the Direction Field again. If a solution "flattens" out along the top or bottom of the graphics window, you may have to change the Axes settings, like trying MinX1=-25 and MaxX1= 25.

### USING PARAMETERS IN YOUR D.E.'S

You may include up to 6 parameters when you enter your D.E.'s. This allows you to change coefficients in the D.E. without having to edit and change the functions every time. The parameter names are a,b,c,d,e,f. For example, the first order D.E. for the current I in a circuit at time t is

$$L \frac{dI}{dt} + RI = k \sin(wt)$$

Where L=inductance, R=resistance,

k=amplitude, w=frequency of the voltage.

To set this up for DEQ, first write this as

$$\frac{dI}{dt} = -\frac{R}{L} I + \frac{k}{L} \sin(wt)$$

Next, replace the dependent variable I with x1, R=a, L=b, k=c, w=d to get  

$$X1'(t) = -(a/b)*x1 + (c/b)*\sin(d*t)$$

Click on Equations and enter the right side of this D.E.  
 Click on Edit|Parameters and enter something like a=10, b=0.5, c=100, d=40.  
 With these values for the parameters, you are looking at a circuit with resistance a=10 Ohms, and inductance b=0.5 henrys, and amplitude c=100 and voltage frequency d=40cps. Now enter an initial condition and plot the current, x1 vs time t. If you want, you can return to the parameters selection and change one or more parameters, for example, change the value a for a different resistance, then Click on Draw to plot the current x1 for that value of resistance. This enables you to investigate solutions to any circuit whose current is governed by this D.E.

Play with this circuit problem some more. Set your Axes with  $t$  from 0 to 1,  $x_1$  from -6 to 6. Try the initial condition  $x_1(0)=0$ . When you plot the solution it will be "jagged", because the frequency of the voltage source is 40cps, so its period is about 0.16 sec. To smooth out your solution, decrease the step size to 0.001. You will be plotting 1000 points, so be patient.

To learn more about other features, click on Help and read about entering functions, saving your graphs, etc. Play around with some of the features until you understand the software. When you have finished, Click on File|Exit to end the session.